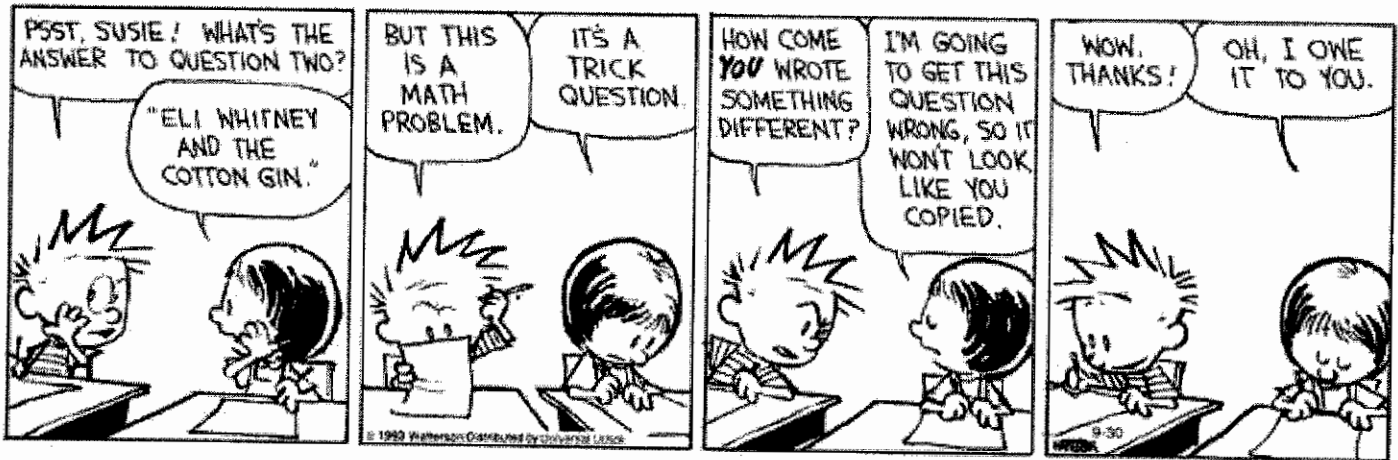


Algebra II - Probability Notes




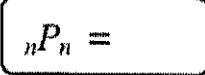


- FUNDAMENTAL COUNTING PRINCIPLE
- COMBINATIONS AND PERMUTATIONS
 - THEORETICAL PROBABILITY
 - CONDITIONAL PROBABILITY
 - PASCAL'S TRIANGLE
- BINOMIAL THEOREM AND PROBABILITY
 - MEAN, MEDIAN, MODE

Name: _____

Permutations and Combinations

Notes

FUNDAMENTAL COUNTING PRINCIPLE	If decision M can be made _____ ways and decision N can be made _____ ways, then the two decisions can be made _____ ways.
EXAMPLES	<ol style="list-style-type: none"> 1. The ice cream shop offers a choice of a 3 cone sizes, 15 flavors, and 8 toppings. How many cones are possible if you can only choose one flavor and one topping? 2. Virginia license plates consist of three letters followed by 4 digits. How many different license plates are possible? 3. Nick's science test has a section with 12 true or false questions. How many different ways can Nick answer these questions? 4. To enter their home, the Clayton family enters a 4-digit code. How many codes are possible?
FACTORIALS 	The _____ of all natural numbers from _____ to _____. Examples: $9! = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$ $12! = \underline{\hspace{2cm}} \quad 8! = \underline{\hspace{2cm}} \quad \frac{17!}{11!} = \underline{\hspace{2cm}}$ $*0! = \underline{\hspace{2cm}}$
PERMUTATIONS 	An arrangement or line up of objects in which _____! Permutation Formula: ${}_n P_r =$  n = total number of objects available r = number of objects to use for the arrangement *Important Shortcut: ${}_n P_n =$ 
EXAMPLES	<ol style="list-style-type: none"> 5. How many ways can you arrange the letters in the word DINOSAUR? 6. Seven students are competing in a geography bee. How many ways can they finish in first, second, and third place?

Permutations and Combinations

Notes

	<p>7. Melanie is taking four classes this semester: American History, Algebra 2, AP English, and Chemistry. How many ways can these four classes be arranged on her schedule?</p> <p>8. There are 16 players on the baseball team. How many ways can the coach make a 9-player batting order?</p> <p>9. There are 28 students in Mr. Miller's homeroom. How many ways can the students elect a student council representative and alternate?</p>	
<p>PERMUTATIONS <i>with Repetition</i></p>	<p>10. How many different 10-letter arrangements are possible using the letters in the word AUTOMOBILE?</p>	<p>11. How many different 9-letter arrangements are possible using the letters in the word DISAPPEAR?</p>
<p>COMBINATIONS</p> <div style="border: 1px solid black; width: 80px; height: 30px; margin: 10px auto;"></div>	<p>A group of objects in which _____</p> <p>Combination Formula: ${}_n C_r =$</p> <p style="text-align: center;">n = total number of objects available r = number of objects to use for the arrangement</p> <p>*Important Shortcut: ${}_n C_n =$</p>	
<p>EXAMPLES</p>	<p>12. Natalie has 16 close friends. How many ways can she choose 5 to be bridesmaids in her wedding?</p> <p>13. There are 24 students in Kyle's kindergarten class. How many ways can he choose eight to attend his birthday party?</p> <p>14. Abby is adopting kittens from the pet store. If there are 18 kittens, how many ways can she choose two?</p> <p>15. There are twelve employees at the sub shop. How many ways can the manager choose four for the Sunday evening shift?</p> <p>16. There are 85 players on the football team. How many ways can the coach choose three to represent the team in the coin toss?</p>	

Permutation or Combination?

Directions: Recall that a **permutation** is an arrangement with a specific order, while a **combination** is a group with no specific order. Determine whether the example represents a permutation or combination, then solve.

1	Example	P or C?	Answer
	How many ways can 4 candy bars be chosen from a store that sells 30 candy bars?		
	How many ways can 13 students line up for lunch?		
	How many ways can you make a 3-letter arrangements out of the letters in the word TRAPEZOID .		
	How many ways can you choose 2 books from a shelf of 40 books?		
	How many ways can 12 swimmers finish in first, second, and third place?		
	How many ways can Mrs. Sullivan choose two students from 27 to help put away calculators at the end of class?		
	You have enough tickets to play 6 different games at the amusement park. If there are 14 games, how many ways can you choose six?		
	How many different ways can 9 trumpet players in the marching band line up?		
	Seven students worked together on a project. How many ways can their teacher choose four to present the project?		
	There are 18 offensive players on the hockey team. How many ways can the coach choose a left wing, center, and right wing to start the game?		
	How many different 12-letter arrangements can be made using the letters in the word INDIANAPOLIS ?		
	There are 26 gold fish in the tank at the store. How many ways can Ben choose five?		

Name: _____

Unit 11: Probability & Statistics

Date: _____ Bell: _____

Homework 1: Fundamental Counting Principle, Permutations & Combinations

**** This is a 2-page document! ****

Directions: Find the total number of outcomes that are possible.	
<p>1. In a school building, there are 8 exterior doors and 12 stairways to enter the second floor. Find the total number of ways to reach the second floor.</p>	<p>2. Kendall went shopping and purchased 7 shirts, 3 pairs of pants, 2 jackets. How many different outfits are possible?</p>
<p>3. An internet passcode consists of a digit followed by a letter, followed by another digit. Assuming the digits are 0-9. How many different passcodes are possible?</p>	<p>4. Grant is rolling a standard six-sided number die eight times. How many outcomes are possible?</p>
<p>5. A quiz has eight multiple choice questions with four options for each (A, B, C, and D). How many ways are there to answer the questions?</p>	<p>6. When Jack bought his new truck, there were 96 different ways his truck could be equipped. He had four choices of engines and two choices of transmissions. If the only other choice was color, how many colors were available?</p>

Directions: Evaluate the following.			
7. $6!$	8. $\frac{16!}{7!}$	9. $\frac{7!}{10!}$	10. $\frac{2! \cdot 9!}{(10-7)!}$
11. ${}_{20}P_4$	12. ${}_{15}C_8$	13. ${}_{17}P_7$	14. ${}_{27}C_{22}$
<p>15. Find ${}_9P_9$ and ${}_9C_9$. Why do the answers differ? Explain.</p> <hr/> <hr/> <hr/>			

Directions: Determine whether the situations represent permutations or combinations. Then solve.		
Example	P/C	Solve
16. Scott has 7 chores to complete this Saturday. How many ways can he arrange the order in which he does them?		
17. There are 16 different colored markers in the bucket. How many ways can Kelly choose five of them?		
18. How many ways are there to elect a President, Vice President, Secretary, and Treasurer, from a club with 32 members?		
19. The dance company is choosing 3 new dancers from a group of 25 who try out. How many ways can they choose the new dancers?		
20. There are ten toppings available to make an ice cream sundae. How many ways can Max choose two?		
21. How many different 11-letter arrangements can be made using the letters in the word FIRECRACKER ?		
22. Mindy is purchasing songs from an album on iTunes. If the album has 14 songs, how many ways can she choose 6?		
23. Carl is choosing a four-character password for his cell phone. The password can contain letters and digits only, and can't include repeating characters. How many passwords are possible?		
24. Mr. Pratt has 26 students in his math class. He has three prizes to give away: a pencil, an eraser, and a no homework coupon. How many ways can he choose three students to win these awards?		
25. How many ways are there to choose seven cards from a deck of 52?		

Theoretical Probability

Notes

Theoretical Probability

- Probability is the measure of how _____ an event is to occur.
- The set of all possible outcomes is called the **sample space**.
- For equally likely outcomes, the theoretical probability of an event, $P(E)$, is the _____ of the number of favorable outcomes to the total number of outcomes possible.

Simple Events

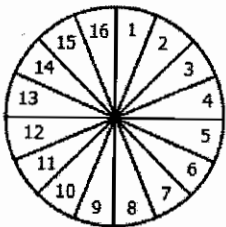
The probability of one event.

- | | |
|---|---|
| 1. A jar contains 32 red marbles and 28 blue marbles. What is the probability that a randomly selected marble is red? | 2. A letter in the word RESTORATION is randomly selected. What is the probability of selecting a vowel? |
| 3. A day in the month of January is randomly selected. What is the probability of selecting a prime number? | 4. Two dice are rolled. What is the probability that the sum of the two dice on the next roll is at least 9? |
| 5. What is the probability of drawing a heart or a club from a standard deck of cards? | 6. There are 8 books lettered A-H on the shelf. If Scott randomly chose two books, what is the probability that he chose books A and B? |

Complement of an Event

The complement of an event is the probability of the event _____ happening. Since the sum of all probabilities in sample space is _____, the probability of an event not happening is $P(\sim E) =$ _____.

- | | |
|--|--|
| 7. The probability that it will snow tomorrow is $7/20$. What is the probability that it will not snow? | 8. A month of the year is randomly selected. What is the probability of getting a month that does not begin with the letter A? |
| 9. If the spinner to the left is spun, find the probability that it lands on a number that is not prime. | 10. Two dice are rolled. What is the probability of not getting doubles? |



Theoretical Probability

Notes

Compound Events

The probability of two or more simple events.

Independent Events

When the outcome of one event does not affect the outcome of the other event.

$$P(A \text{ and } B) =$$

Dependent Events

When the outcome of one event does affect the outcome of the other event.

$$P(A \text{ and } B) =$$

Independent Events

11. A die is rolled 3 times. What is the probability of getting 1's on each roll?

12. A coin is tossed, then a day of the week is selected. What is the probability of getting tails then a day starting with the letter T?

A bag contains 8 red crayons, 14 purple crayons, 6 yellow crayons, and 4 green crayons. A crayon is selected, replaced, then another is selected. Find each probability.

13. $P(\text{purple then yellow})$

14. $P(\text{green then red})$

15. $P(\text{two purples})$

16. $P(\text{two yellows})$

Dependent Events

Using the same example from above, assume once a crayon is selected, it is NOT replaced. Find each probability.

17. $P(\text{yellow then red})$

18. $P(\text{purple then green})$

19. $P(\text{two reds})$

20. $P(\text{two greens})$

21. A card is drawn from a standard deck, not replaced, and another is drawn. What is the probability of choosing a heart then a spade?

22. Jack had four Snicker bars and 8 Mars bars. He randomly chose a piece of candy, ate it, then chose another. What is the probability that both candy bars were Snickers?



Name: _____

Unit 11: Probability & Statistics

Date: _____ Bell: _____

Homework 2: Theoretical Probability

**** This is a 2-page document! ****

Part I: Simple Probability

Use for questions 1-3: A random two-digit number (10-99) is drawn. Find each probability.

1. $P(32)$	2. $P(\text{odd number})$	3. $P(\text{a multiple of } 5)$
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Use for questions 4-6: A letter is randomly chosen from the word **CANDLESTICK**. Find each probability.

4. $P(\text{a vowel})$	5. $P(N \text{ or } S)$	6. $P(\text{not } C)$
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7. Three coins are tossed. Find the probability that two land on heads.	8. A month is randomly chosen. What is the probability that the month chosen has less than 31 days?
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9. What is the probability of drawing a 9 or diamond from a standard deck of cards?	10. Credit cards place a three-digit security code on the back of cards. What is the probability that a code starts with the number 7?
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11. Two dice are rolled. What is the probability of not getting doubles?	12. Mikayla has the following songs on her iPod: 14 Taylor Swift songs, 16 Meghan Trainor songs, and 17 Katy Perry songs. What is the probability that the next song that plays is not Katy Perry?
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Part II: Compound Probability

13. A dice is rolled, then a coin is tossed. What is the probability of getting a 5 then tails?	14. A coin is tossed, then a number 1-10 is chosen at random. What is the probability of getting heads then a number less than 4?
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15. Natalie guessed on the last four true or false questions on her math quiz. What is the probability that she got all four questions correct?

16. A card is drawn from a standard deck and a letter is chosen from the word **INCREDIBLE**. What is the probability of drawing a king then getting an I?

Use for questions 17-20: A bag contains 30 lottery balls numbered 1-30. A ball is selected, replaced, then another is drawn. Find each probability.

17. $P(\text{and even, then odd})$

18. $P(7, \text{ then a number greater than } 16)$

19. $P(\text{a multiple of } 5, \text{ then a prime number})$

20. $P(\text{two even numbers})$

Use for questions 21-24: A bag contains 30 lottery balls numbered 1-30. A ball is selected, NOT replaced, then another is drawn. Find each probability.

21. $P(\text{a 2-digit number, then } 4)$

22. $P(19, \text{ then a multiple of } 4)$

23. $P(24, \text{ then a number less than } 15)$

24. $P(\text{two perfect squares})$

25. A football team has 5 freshman, 8 sophomores, 11 juniors, and 16 seniors. If two are chosen at random to participate in the coin toss, what the probability that both players chosen will be seniors?

26. Ryan's mom randomly chooses two days each week for Ryan to do his chores. What is the probability that she picks Saturday and Sunday?

Conditional Probability

Notes

Conditional Probability

- Given events A and B , conditional probability is the probability of event B occurring, knowing that event A has _____.
- Notation: _____
- Read as: _____

Conditional Probability Formula:

Examples

- Mitchell drew a card from a standard deck of playing cards. What is the probability that he drew a queen, given that the card was red?
- A number from 1-100 is randomly selected. What is the probability that it is a perfect square, given that it is an odd number?
- There are 62 people that take yoga class and 48 people that take spinning class at the gym. Fifteen people that take yoga also take spinning. If a person from these groups is selected at random, find the probability that he or she is takes yoga class, if you know they take spinning class.
- Of the 30 students in Mrs. Smith's math class, 13 are athletes and 9 are in band. Three of the athletes are also in band. If one student is chosen at random, find the probability that this student is in band, if it is known that they are not an athlete.

Two-Way Tables

A table that records data that pertains to two different categories.

Example: The table shows the results of a poll of 50 randomly selected students who were asked whether the prefer to watch hockey or football. Find each probability.

	Boys	Girls
Prefer Football	18	6
Prefer Hockey	10	16

a) $P(\text{prefer football} | \text{girl})$

b) $P(\text{boy} | \text{hockey})$

Conditional Probability

Notes

Directions: Find each probability using the table.

5. The table shows the results of a poll of 150 randomly selected middle school students who were asked if they take French or Spanish.

	French	Spanish
6 th Grade	19	23
7 th Grade	25	12
8 th Grade	38	33

a) $P(\text{a 7th grader who takes French})$

b) $P(\text{8th grader} | \text{takes Spanish})$

c) $P(\text{takes French} | \text{6th grader})$

d) $P(\text{7th grader} | \text{takes French})$

6. The table shows the results of a poll of randomly selected juniors and seniors who were asked if they attended prom.

		Juniors	Seniors
Prom	Yes	28	97
	No	56	19

a) $P(\text{a junior that did not attend prom})$

b) $P(\text{did not attend prom} | \text{senior})$

c) $P(\text{junior} | \text{attended prom})$

Frequencies

- **Joint Relative Frequencies:** The values in each category divided by total number of values.
- **Marginal Relative Frequencies:** Found by totaling each row and column.

Examples

7. The table below shows the number of first class and coach class passengers on a plane who checked one bag, two bags, or no bags. Find the joint relative and marginal relative frequencies, then find each probability.

	First Class	Coach Class	Total
2 bags	16	52	
1 bag	12	60	
0 bags	2	18	
Total			

a) $P(\text{a coach class passenger that did not check any bags})$

b) $P(\text{a first class passenger that checked at least one bag})$

c) $P(\text{coach class} | \text{checked one bag})$

d) $P(\text{checked two bags} | \text{first class})$



Name: _____

Unit 11: Probability & Statistics

Date: _____ Bell: _____

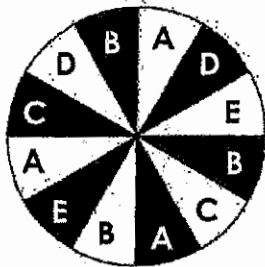
Homework 3: Conditional Probability

**** This is a 2-page document! ****

Use for questions 1-2: A bucket contains 50 lottery balls numbered 1-50. One is drawn at random. Find each probability.

1. $P(\text{multiple of 6} \mid \text{2-digit number})$	2. $P(\text{at least 20} \mid \text{prime number})$
3. Marti rolls two dice. What is the probability that the sum of the dice is 7, given that the first die is showing a 2?	4. Blake randomly chose a letter from alphabet. What is the probability that this letter has at least one line of symmetry, given that it is a consonant?
5. A card is randomly selected from a standard deck of playing cards. Find the probability that it is a face card, given that a black card is drawn.	6. A month of the year is randomly chosen. Find the probability that it has no more than 30 days, given that it starts with the letter A.

Use for 7-9: The wheel below is spun. Find each probability.



7. $P(\text{black} \mid \text{A})$
8. $P(\text{C} \mid \text{white})$
9. $P(\text{black} \mid \text{B or E})$

10. Out of the 125 children at summer camp, 45 signed up for swimming and 38 signed up for arts and crafts. Twelve students who signed up for swimming also signed up for arts and crafts. If a child is randomly selected, what is the probability that they are signed up for swimming, if it is known that they did not sign up for arts and crafts?

11. Out of the 56 players on the football team, 24 are on honor roll and 18 have perfect attendance. Seven who are on honor roll also have perfect attendance. If a player is chosen at random, what is the probability that they are on honor roll, if it is known that they also have perfect attendance?

Directions: Find each probability using the table.

- 12.** The table below shows the number of students that do or do not have their own car and whether they have part time jobs.

		Part Time Job	
		Yes	No
Car	Yes	78	18
	No	30	24

- a) $P(\text{a student with a part-time job without a car})$

- b) $P(\text{no car} | \text{ does not have a part-time job})$

- c) $P(\text{part-time job} | \text{ car})$

- 13.** The table below shows the fate of the first, second, and third class passengers on the Titanic.

	Survived	Died
	1 st Class	199
2 nd Class	117	155
3 rd Class	172	537

- a) $P(\text{a 3rd class survivor})$

- b) $P(\text{1st class} | \text{ died})$

- c) $P(\text{survived} | \text{ 2nd class})$

- d) $P(\text{1^{st}} or 3rd class} | \text{ survived})$

- 14.** The table below represents the GPA of a group of undergrad students and whether or not they will be attending grad school. Find the joint relative and marginal relative frequencies.

		Attending Grad School	
		Yes	No
GPA	≥ 3.0	96	18
	< 3.0	32	54

		Attending Grad School		
		Yes	No	Total
GPA	≥ 3.0			
	< 3.0			
Total				

If a student is chosen at random, find each probability:

- a) $P(\text{not attending grad school})$

- b) $P(< 3.0 \text{ GPA and not attending grad school})$

- c) $P(\text{not attending grad school} | \geq 3.0 \text{ GPA})$

- d) $P(< 3.0 \text{ GPA} | \text{ attending grad school})$

<p>13. A number 1-30 is randomly selected, followed by a letter from the word TARGET. What is the probability of choosing a perfect square then the letter T?</p>	<p>14. Carter is handing out Valentine's Day cards to his friends. He has 8 Batman cards, 12 Superman cards, and 5 Captain American cards in a bag. What is the probability that the first two he chooses are Batman?</p>
<p>Use for questions 15-16: A bag contains 25 red chips, 18 blue chips, and 29 yellow chips. A chip is selected, replaced, then another chip is chosen. Find each probability.</p>	
<p>15. $P(\text{blue then red})$</p>	<p>16. $P(\text{yellow then blue})$</p>
<p>Use for questions 17-18: A box of pizza contains three cheese slices, seven pepperoni slices, and five supreme slices. Mark randomly chooses a slice, eats it, then chooses another. Find each probability.</p>	
<p>17. $P(\text{cheese then supreme})$</p>	<p>18. $P(\text{two pepperonis})$</p>

Topic 3: Conditional Probability

<p>19. Mason randomly chooses a card from a standard deck of playing cards. What is the probability that it is not an ace, given it is a red card?</p>	<p>20. One of the 50 United States is randomly selected. What is the probability that it has two syllables, given it starts with the letter M?</p>									
<p>21. Of the 318 sophomores, 140 are taking Algebra 2 and 102 are taking Chemistry. Twenty-six of those taking Algebra 2 are also taking chemistry. If a sophomore is chosen at random, find the probability that they are taking Algebra 2, if it is known that they do not take Chemistry.</p>										
<p>22. A group of cell phone owners were asked whether they had an Android or apple phone. Use the results in the table to find each probability.</p> <table border="1" data-bbox="261 1667 646 1806"> <tr> <td></td> <td style="background-color: #cccccc;">Male</td> <td style="background-color: #cccccc;">Female</td> </tr> <tr> <td style="background-color: #cccccc;">Android</td> <td>34</td> <td>19</td> </tr> <tr> <td style="background-color: #cccccc;">Apple</td> <td>30</td> <td>42</td> </tr> </table>		Male	Female	Android	34	19	Apple	30	42	<p>a) $P(\text{a male that owns an android})$</p> <hr/> <p>b) $P(\text{apple} \text{male})$</p> <hr/> <p>c) $P(\text{female} \text{android})$</p>
	Male	Female								
Android	34	19								
Apple	30	42								

23. The table below shows the number of gold, silver, and bronze medals won by the United States and China during the 2012 summer Olympics. Find the joint relative and marginal relative frequencies

	U.S.	China
Gold	46	38
Silver	28	28
Bronze	29	22

	U.S.	China	Total
Gold			
Silver			
Bronze			
Total			

If a medal from those above is randomly selected, find each probability.

a) $P(\text{a U.S. bronze medal})$

b) $P(\text{a gold or silver medal})$

c) $P(\text{silver} \mid \text{China})$

d) $P(\text{U.S.} \mid \text{gold})$

e) $P(\text{China} \mid \text{not silver})$

f) $P(\text{bronze} \mid \text{not China})$

Topic 4: Binomial Probability & Binomial Theorem

Use for questions 24-27: The soda company is printing prizes on the inside of their bottle caps, with a 3 in 5 chance of winning. If Tom purchases a 12-pack of soda, find each probability.

24. $P(\text{exactly two prizes})$

25. $P(\text{exactly five prizes})$

26. $P(\text{at least ten prizes})$

27. $P(\text{no more than three prizes})$